Revista Colombiana de Matemáticas
Volumen 52(2018)2, páginas 161-169

Inductive lattices of totally composition formations

Retículos inductivos de formaciones totalmente compositivas

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Abstract. Let τ be a subgroup functor such that all subgroups of every finite group G contained in $\tau(G)$ are subnormal in G. In this paper, we give a simple proof of the fact that the lattice of all τ-closed totally composition formations of finite groups is inductive.

Key words and phrases. Finite group, formation of groups, satellite of formation, τ-closed totally composition formation, inductive lattice of formations.

2010 Mathematics Subject Classification. Primary 20F17, secondary 20D10.

Resumen. Sea τ un funtor de subgrupo de modo que todos los subgrupos de cualquier grupo finito G contenido en $\tau(G)$ son subnormales en G. En este artículo, damos una demostración simple de que el retículo de todas las formaciones de composición totalmente τ-cerradas de los grupos finitos es inductivo.

Palabras y frases clave. Grupo finito, formación de grupos, satélite de formación, formación de composición totalmente τ-cerrada, retículo inductivo de formaciones.

1. Introduction

All groups considered in this paper are finite. A class of groups is a collection of groups satisfying the property that if a group G belongs to the collection, then every group isomorphic to G is also in the collection.

If a class of groups is a *formation*, it is closed with respect to forming quotient groups and subdirect products. This notion introduced by Gaschütz [3] in 1963 immediately became an object of extensive investigations. Saturated formations are very important in group theory; composition formations form a broader family of formations. By Baer's theorem, composition formations are precisely solvably saturated formations [2, p. 373].

Skiba [10] introduced the concept of an inductive lattice of formations in order to adapt lattice-theoretical methods for the investigation of saturated formations. This concept plays an important role in the research of the lattices of formations and their law systems (see Chapter 4 of the book [10], Chapter 4 of the book [19]; and the papers [5, 6, 7, 8, 12, 13, 14, 18, 20, 21]).

Let Θ be a complete lattice of formations. A satellite f is called Θ-*valued* if all its values belong to Θ. We denote by Θ^c the set of all formations having a composition Θ-valued satellite. In [11, p. 901], it is shown that this set is a complete lattice of formations.

A complete lattice Θ^c is called *inductive* if for any collection of formations $\{\mathfrak{F}_i = CLF(f_i) \mid i \in I\}$, where f_i is an integrated satellite of $\mathfrak{F}_i \in \Theta^c$, the following equality holds:

$$\vee_{\Theta^c}(\mathfrak{F}_i \mid i \in I) = CLF(\vee_{\Theta}(f_i \mid i \in I)).$$

The inductance of a lattice Θ^c, in fact, means that a research of the operation $\vee_{\Theta^c}$ on the set Θ^c can be reduced to a research of the operation $\vee_{\Theta}$ on the set Θ. Therefore, the inductance is one very useful property of the lattice Θ^c.

Vorob'ev [17] proved that the lattice of all totally saturated formations is inductive. Moreover, it is already known that the lattice of all multiply composition formations is inductive (see [16]). However, the following question was still open.

Question. *Is the lattice of all totally composition formations inductive?*

The aim of the present paper is to give a simple proof of the following theorem which gives a positive answer to this question.

Theorem 1.1. *The lattice of all τ-closed totally composition formations c_∞^τ is inductive.*

2. Terminologies and notations

All unexplained notations and terminologies are standard. The reader is referred to [1, 2, 4, 11] if necessary.

2.1. Subgroup functor τ

In various applications of the theory of classes of finite groups, it is often necessary to use formations closed with respect to some subgroup systems. Skiba [10] introduced the concept of a subgroup functor, which covers all the systems of subgroups under consideration.

In each group G, we select a system of subgroups $\tau(G)$. We say that τ is a *subgroup functor* if (1) $G \in \tau(G)$ for every group G; (2) for every epimorphism

$\varphi : A \to B$, and each $H \in \tau(A)$ and $T \in \tau(B)$, we have $H^{\varphi} \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$.

If $\tau(G) = \{G\}$, then the functor τ is called *trivial*. A formation $\mathfrak{F}$ is called *τ-closed* if $\tau(G) \subseteq \mathfrak{F}$ for every group G of $\mathfrak{F}$ (see [10]).

We consider only subgroup functors τ such that for every group G all subgroups of $\tau(G)$ are subnormal in G.

2.2. Composition formations

The set of all primes is denoted by $\mathbb{P}$. Let $p \in \mathbb{P}$, and G a group. Then the subgroup $C^p(G)$ is the intersection of the centralizers of all the abelian p-chief factors of G, with $C^p(G) = G$ if G has no abelian p-chief factors.

For every collection of groups $\mathfrak{X}$, we write $\mathrm{Com}(\mathfrak{X})$ to denote the class of all groups L such that L is isomorphic to some abelian composition factor of some group in $\mathfrak{X}$. If $\mathfrak{X}$ is the set of one group G, then we write $\mathrm{Com}(G)$ instead of $\mathrm{Com}(\mathfrak{X})$.

The symbol $R(G)$ denotes the product of all solvable normal subgroups of G. We consider a function f of the form

$$f : \mathbb{P} \cup \{0\} \to \{\text{formations of groups}\}, \qquad (*)$$

and the class of groups

$$CLF(f) = (G \mid G/R(G) \in f(0);\ G/C^p(G) \in f(p) \text{ for all } p \in \pi(\mathrm{Com}(G))).$$

If $\mathfrak{F}$ is a formation such that $\mathfrak{F} = CLF(f)$ for a function f of the form $(*)$, then $\mathfrak{F}$ is said to be *composition* (solvably saturated) formation, and f is said to be a *composition satellite* of $\mathfrak{F}$ (see [4, p. 4]).

If the values of composition satellites of some formation are themselves composition formations, then this circumstance leads to the following natural definition. Every formation is 0-multiply composition; for $n > 0$, a formation $\mathfrak{F}$ is called *n-multiply composition* if $\mathfrak{F} = CLF(f)$, and all nonempty values of f are $(n-1)$-multiply composition formations (see [11]).

A formation is called *totally composition* if it is n-multiply composition for all positive integers n.

2.3. Lattices of formations

A set of formations Θ is called a *complete lattice of formations* if the intersection of every set of formations in Θ belongs to Θ, and there is a formation $\mathfrak{F}$ in Θ such that $\mathfrak{M} \subseteq \mathfrak{F}$ for every other formation $\mathfrak{M}$ of Θ (see [10]).

Every complete lattice of formations is a complete lattice in the ordinary sense. Various collections of formations form complete lattices; for example, the set of all saturated formations [10, p. 151], and the set of all composition (solvably saturated) formations [9, p. 97] are complete lattices of formations. Moreover for all positive integers n, the set of all n-multiply composition formations c_n, and the set of all totally composition formations $c_\infty = \bigcap_{n=1}^{\infty} c_n$ are complete lattices of formations (see [11, p. 904]).

A formation in Θ is called a Θ-*formation*. Let Θ be a complete lattice of formations, and let $\{\mathfrak{F}_i \mid i \in I\}$ be an arbitrary collection of Θ-formations. We denote

$$\vee_\Theta(\mathfrak{F}_i \mid i \in I) = \Theta\mathrm{form}(\bigcup_{i\in I} \mathfrak{F}_i).$$

In particular, we write $\vee_\infty^\tau(\mathfrak{F}_i \mid i \in I) = c_\infty^\tau\mathrm{form}(\cup_{i\in I}\mathfrak{F}_i)$.

If $\mathfrak{M}, \mathfrak{H} \in \Theta$, then $\mathfrak{M}\bigcap\mathfrak{H}$ is the greatest lower bound for $\{\mathfrak{M}, \mathfrak{H}\}$ in Θ; and $\mathfrak{M} \vee_\Theta \mathfrak{H}$ is the least upper bound for $\{\mathfrak{M}, \mathfrak{H}\}$ in Θ.

Let $\{f_i \mid i \in I\}$ be a collection of Θ-valued functions of the form $(*)$. Then by $\vee_\Theta(f_i \mid i \in I)$ we denote a function f such that

$$f(a) = \Theta\mathrm{form}(\cup_{i\in I} f_i(a))$$

for all $a \in \mathbb{P} \cup \{0\}$.

3. Preliminaries

Following the paper [11], we set for every collection of groups $\mathfrak{X}$:

$$\mathfrak{X}(C^p) = \begin{cases} \mathrm{form}(G/C^p(G) \mid G \in \mathfrak{X}) & \text{if } p \in \pi(\mathrm{Com}(\mathfrak{X})); \\ \varnothing & \text{if } p \in \mathbb{P} \setminus \pi(\mathrm{Com}(\mathfrak{X})). \end{cases}$$

We recall that the symbol $\mathfrak{N}_p$ denotes the class of all p-groups. Let $\mathfrak{F} = CLF(F)$, where $F(0) = \mathfrak{F}$ and $F(p) = \mathfrak{N}_p\mathfrak{F}(C^p)$ for all $p \in \mathbb{P}$. Then the satellite F is called a *canonical composition satellite* of the formation $\mathfrak{F}$. By [11, Remark 1], every composition formation possesses a canonical composition satellite.

Lemma 3.1. [11, Lemma 8] *Let Θ be a complete lattice of formations such that $\Theta^c \subseteq \Theta$ and let the formation $\mathfrak{N}_p\mathfrak{H}$ belongs to Θ for each formation $\mathfrak{H} \in \Theta$ and every prime p. If $\mathfrak{F} = CLF(F) \in \Theta^c$, then the satellite F is Θ-valued.*

Lemma 3.2. [16, Lemma 1] *Let n be a positive integer. Then we have*

$$(c_{n-1}^\tau)^c = c_n^\tau.$$

Corollary 3.3. *The following equality holds: $(c_\infty^\tau)^c = c_\infty^\tau$.*

Proof. The inclusion $(c_\infty^\tau)^c \subseteq c_\infty^\tau$ is obvious. Let $\mathfrak{F} \in c_\infty^\tau$ and F be a canonical composition satellite of $\mathfrak{F}$. Then by Lemmas 3.1 and 3.2 for all $a \in \mathbb{P} \cup \{0\}$ and each positive integer n, the formation $F(a)$ is τ-closed n-multiply composition. Thus, the satellite F is c_∞^τ-valued. Consequently, $\mathfrak{F} \in (c_\infty^\tau)^c$, and we have $c_\infty^\tau \subseteq (c_\infty^\tau)^c$. ☑

Lemma 3.4. [22, Lemma 2.1] *Let $\mathfrak{F} = CLF(F)$ be a τ-closed n-multiply composition formation, where n is a positive integer. Then the satellite F is c_n^τ-valued.*

From Lemma 3.4 follows the corollary.

Corollary 3.5. *Let $\mathfrak{F} = CLF(F)$ be a τ-closed totally ω-composition formation. Then the satellite F is c_∞^τ-valued.*

Let $\{f_i \mid i \in I\}$ be a collection of composition satellites. Then by $\bigcap_{i\in I} f_i$, we denote the composition satellite f such that $f(a) = \bigcap_{i\in I} f_i(a)$ for all $a \in \mathbb{P}\cup\{0\}$ (see [11]).

Lemma 3.6. [11, Lemma 2] *Let $\mathfrak{F} = \bigcap_{i\in I} \mathfrak{F}_i$, where $\mathfrak{F}_i = CLF(f_i)$. Then $\mathfrak{F} = CLF(f)$, where $f = \bigcap_{i\in I} f_i$.*

Let $\{f_i \mid i \in I\}$ be the collection of all composition c_∞^τ-valued satellites of a formation $\mathfrak{F}$. Since the lattice c_∞^τ is complete using Lemma 3.6, we conclude that $f = \bigcap_{i\in I} f_i$ is a composition c_∞^τ-valued satellite of $\mathfrak{F}$. The satellite f is called *minimal*.

Let Θ be a complete lattice of formations. Then $\Theta\mathrm{form}\mathfrak{X}$ is the intersection of all Θ-formations containing a collection of groups $\mathfrak{X}$. Thus, $c_\infty^\tau\mathrm{form}\mathfrak{X}$ is the intersection of all τ-closed totally composition formations containing a collection of groups $\mathfrak{X}$. The next lemma immediately follows from [11, Lemma 5] by Corollary 3.3, and gives a description of the minimal c_∞^τ-valued satellite of a formation $c_\infty^\tau\mathrm{form}\mathfrak{X}$.

Lemma 3.7. *Let $\mathfrak{X}$ be a nonempty collection of groups, $\mathfrak{F} = c_\infty^\tau\mathrm{form}\mathfrak{X}$, $\pi = \pi(\mathrm{Com}(\mathfrak{X}))$, and let f be the minimal c_∞^τ-valued composition satellite of $\mathfrak{F}$. Then the following statements hold:*

1) $f(0) = c_\infty^\tau\mathrm{form}(G/R(G) \mid G \in \mathfrak{X})$;
2) $f(p) = c_\infty^\tau\mathrm{form}(G/C^p(G) \mid G \in \mathfrak{X})$ *for all* $p \in \pi$;
3) $f(p) = \varnothing$ *for all* $p \in \mathbb{P} \setminus \pi$;
4) *if $\mathfrak{F} = CLF(h)$ and the satellite h is c_∞^τ-valued, then for all $p \in \pi$ we have*

$$f(p) = c_\infty^\tau\mathrm{form}(G \mid G \in h(p) \cap \mathfrak{F} \text{ and } O_p(G) = 1), \text{ and}$$
$$f(0) = c_\infty^\tau\mathrm{form}(G \mid G \in h(0) \cap \mathfrak{F} \text{ and } R(G) = 1).$$

By Lemma 3.7, it is easy to show the following assertion.

Corollary 3.8. *Let f_1 and f_2 be the minimal composition c_∞^τ-valued satellites of formations $\mathfrak{F}_1$ and $\mathfrak{F}_2$ respectively. Then $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ if and only if $f_1 \leq f_2$.*

If $\mathfrak{F} = CLF(f)$ and $f(a) \subseteq \mathfrak{F}$ for all $a \in \mathbb{P} \cup \{0\}$, then f is called an *integrated* satellite of $\mathfrak{F}$.

4. Inductance of the lattice $c^{\tau}_{\omega_\infty}$

Proof of Theorem. Let $\{\mathfrak{F}_i \mid i \in I\}$ be a collection of τ-closed totally composition formations, and f_i be an integrated c^{τ}_{∞}-valued composition satellite of $\mathfrak{F}_i$. Let

$$\mathfrak{F} = CLF(f) = \vee^{\tau}_{\infty}(\mathfrak{F}_i \mid i \in I), \text{ and } \mathfrak{M} = CLF(\vee^{\tau}_{\infty}(f_i \mid i \in I)).$$

We shall show that $\mathfrak{F} = \mathfrak{M}$ proceeding by induction on i.

Step 1. Let $i = 2$, $p \in \mathbb{P}$, and h_j be the minimal c^{τ}_{∞}-valued composition satellite of the formation $\mathfrak{F}_j = CLF(f_j)$, where $j = 1, 2$. Then by Corollary 3.5, we have

$$h_j(p) \subseteq f_j(p) \subseteq \mathfrak{N}_p h_j(p) = F_j(p) \in c^{\tau}_{\infty},$$

where F_j is the canonical c^{τ}_{∞}-valued composition satellite of the formation $\mathfrak{F}_j$. Let $\mathfrak{F} = CLF(F)$, where F is the canonical c^{τ}_{∞}-valued composition satellite of the formation $\mathfrak{F}$. Then by Lemma 3.7, we have

$$h(p) = c^{\tau}_{\infty}\text{form}((\mathfrak{F}_1 \cup \mathfrak{F}_2)(C^p)) = c^{\tau}_{\infty}\text{form}(\mathfrak{F}_1(C^p) \cup \mathfrak{F}_2(C^p)) =$$
$$c^{\tau}_{\infty}\text{form}(h_1(p) \cup h_2(p)) \subseteq f(p) \subseteq$$
$$\mathfrak{N}_p c^{\tau}_{\infty}\text{form}(h_1(p) \cup h_2(p)) = \mathfrak{N}_p h(p) = F(p).$$

Thus, we have $h(p) \subseteq f(p) \subseteq F(p)$ for all $p \in \mathbb{P}$; moreover, it holds $h(0) \subseteq f(0) \subseteq F(0)$. Hence, $h(a) \subseteq f(a) \subseteq F(a)$ for all $a \in \mathbb{P} \cup \{0\}$ implies $h \leq f \leq F$. Consequently, we have $\mathfrak{F}_1 \vee^{\tau}_{\infty} \mathfrak{F}_2 = CLF(f_1 \vee^{\tau}_{\infty} f_2)$.

Step 2. Let $i > 2$, and the assertion is true for $i = r - 1$ by induction. Then $\mathfrak{F}_1 \vee^{\tau}_{\infty} \dots \vee^{\tau}_{\infty} \mathfrak{F}_{r-1} = CLF(f_1 \vee^{\tau}_{\infty} ... \vee^{\tau}_{\infty} f_{r-1})$. By Step 1, we have

$$\mathfrak{F} = c^{\tau}_{\infty}\text{form}((\mathfrak{F}_1 \vee^{\tau}_{\infty} ... \vee^{\tau}_{\infty} \mathfrak{F}_{r-1}) \cup \mathfrak{F}_r) = CLF(f),$$

where

$$f(a) = c^{\tau}_{\infty}\text{form}((f_1(a) \vee^{\tau}_{\infty} ... \vee^{\tau}_{\infty} f_{r-1}(a)) \cup f_r(a)) =$$
$$f_1(a) \vee^{\tau}_{\infty} ... \vee^{\tau}_{\infty} f_r(a) = (f_1 \vee^{\tau}_{\infty} ... \vee^{\tau}_{\infty} f_r)(a)$$

for each $a \in \mathbb{P} \cup \{0\}$. Therefore, we have $f = f_1 \vee^{\tau}_{\infty} ... \vee^{\tau}_{\infty} f_r$. This proves the theorem. ☑

Each complete sublattice of the inductive lattice is an inductive lattice. Thus, we have the following result.

Corollary 4.1. *Let θ be a complete sublattice of the lattice c^{τ}_{∞}. Then θ is inductive.*

If τ is trivial, we have the following result.

Corollary 4.2. *The lattice of all totally composition formations is inductive.*

Corollary 4.3. *The lattice of all solvable totally composition formations is inductive.*

By Lemma 3.6, we have the following corollary.

Corollary 4.4. *Let* $\xi(x_1, ..., x_m)$ *be a term of signature* $\{\cap, \vee_\infty^\tau\}$*, and let* f_i *be an integrated* c_∞^τ*-valued composition satellite of a formation* $\mathfrak{F}_i$*, where* $i = 1, ..., m$*. Then, we have* $\xi(\mathfrak{F}_1, ..., \mathfrak{F}_m) = CLF(\xi(f_1, ..., f_m))$*.*

5. Some applications

Let A be a group, and p be a prime. We use $Z_p \wr A$ to denote the regular wreath product of groups Z_p and A (see [2, p. 66]).

Lemma 5.1. *Let* $\mathfrak{F}_i = c_\infty^\tau\mathrm{form}(Z_p \wr A_i)$*, where* $p \notin \pi(A_i)$ *for* $i = 1, 2$*. Then* $f(p) = f_1(p) \cap f_2(p)$*, where* f_i *and* f *are the minimal* c_∞^τ*-valued composition satellites of the formations* $\mathfrak{F}_i$ *and* $\mathfrak{F} = \mathfrak{F}_1 \cap \mathfrak{F}_2$*, respectively.*

Proof. See proof of [15, Lemma 3.1]. ☑

The following lemma is proved by direct calculation.

Lemma 5.2. *Let* f_i *be the minimal* c_∞^τ*-valued composition satellite of a formation* $\mathfrak{F}_i$*, where* $i \in I$*. Then* $\vee_\infty^\tau(f_i \mid i \in I)$ *is the minimal* c_∞^τ*-valued composition satellite of the formation* $\mathfrak{F} = \vee_\infty^\tau(\mathfrak{F}_i \mid i \in I)$*.*

Proposition 5.3. *Let* $\mathfrak{F}_i = c_\infty^\tau\mathrm{form}(Z_p \wr A_i)$ *for* $p \notin \pi(A_i)$*, where* $i = 1, \ldots, m$*. Let* f_i *be the minimal* c_∞^τ*-valued composition satellite of* $\mathfrak{F}_i$ *and* $f(p) = \xi(f_1, \ldots, f_m)(p)$*, where* $\xi(x_1, \ldots, x_m)$ *is a term of signature* $\{\cap, \vee_\infty^\tau\}$*. Then* f *is the minimal* c_∞^τ*-valued composition satellite of the formation* $\mathfrak{F} = \xi(\mathfrak{F}_1, \ldots, \mathfrak{F}_m)$*.*

Proof. Let $h = \xi(f_1, \ldots, f_m)$. By Corollary 4.4, we have

$$\xi(\mathfrak{F}_1, \ldots, \mathfrak{F}_m) = CLF(h).$$

We shall show that $h(p) = f(p)$ by induction on the number r of occurrences of the symbols in $\{\cap, \vee_\infty^\tau\}$ into ξ.

The case $r = 1$ holds using Lemmas 5.1 and 5.2.

Let the term ξ have $r > 1$ occurrences of the symbols in $\{\cap, \vee_\infty^\tau\}$. Let ξ have the form $\xi(x_1, \ldots, x_m) = \xi_1(x_{i_1}, \ldots, x_{i_a}) \triangle \xi_2(x_{j_1}, \ldots, x_{j_b})$, where $\{x_{i_1}, \ldots, x_{i_a}\} \cup \{x_{j_1}, \ldots, x_{j_b}\} = \{x_1, \ldots, x_m\}$, and $\triangle \in \{\cap, \vee_\infty^\tau\}$. We suppose that the assertion is true for the terms ξ_1 and ξ_2. By induction, we have $h_1(p) = \xi_1(f_{i_1}, \ldots, f_{i_a})(p)$ and $h_2(p) = \xi_2(f_{j_1}, \ldots, f_{j_b})(p)$, where h_1 and h_2 are the minimal c_∞^τ-valued composition satellites of the formations $\xi_1(\mathfrak{F}_{i_1}, \ldots, \mathfrak{F}_{i_a})$ and $\xi_2(\mathfrak{F}_{j_1}, \ldots, \mathfrak{F}_{j_b})$, respectively. Thus, we have

$$f(p) = h_1(p) \triangle h_2(p) =$$

$$\xi_1(f_{i_1}(p), \dots, f_{i_a}(p)) \triangle \xi_2(f_{j_1}(p), \dots, f_{j_b}(p)) =$$
$$\xi(f_1(p), \dots, f_m(p)) = \xi(f_1, \dots, f_m)(p) = h(p),$$

as claimed. ☑

Acknowledgement

The author is grateful to the referees for a number of suggestions which have improved this paper.

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(Recibido en marzo de 2018. Aceptado en mayo de 2018)

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