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Existential Graphs on nonplanar surfaces

Gráficos existenciales sobre superficies no planas

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Abstract. Existential graphs on the plane constitute a two-dimensional representation of classical logic, in which a Jordan curve stands for the negation of its inside. In this paper we propose a program to develop existential Alpha graphs, which correspond to propositional logic, on various surfaces. The geometry of each manifold determines the possible Jordan curves on it, leading to diverse interpretations of negation. This may open a way for appointing a "natural" logic to any surface.

Key words and phrases. Existential graphs, 2-dimensional topological manifolds, Jordan's Curve Theorem.

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Resumen. Los gráficos existenciales sobre el plano constituyen una representación bidimensional de la lógica clásica, en la cual una curva de Jordan indica la negación de su interior. En este artículo se propone un programa para desarrollar los gráficos existenciales Alfa, que corresponden a la lógica proposicional, sobre diferentes superficies. La geometría de cada variedad determina las posibles curvas de Jordan sobre ella, lo cual conduce a interpretaciones diversas de la negación. Esto puede abrir el camino para asignar una lógica "natural" a cualquier superficie.

Palabras y frases clave. Gráficos existenciales, 2-variedades topológicas, teorema de la curva de Jordan.

1. Introduction

Existential graphs, which were considered by his creator Charles S. Peirce his *chef d'œvre* [11], may be seen as a geometric representation of mathematical logic. In this system, logical formulas are laid out as two-dimensional diagrams on the plane and inference is attained by appropriate graphical transformations on this figures. Just as geometry changes when the plane is replaced by a different surface, if existential graphs are drawn on a non planar surface the outcoming system most surely corresponds to some non classical logic. This paper is an incipient inquiry into the general features of systems of existential graphs on surfaces different from the plane. Possibly, it is also the first time that Geometry returns through existential graphs to Logic, if not results at least some genuine mathematical problems.

In this first approach we will consider only propositional logic, which is expressed graphically by the subsystem of existential graphs called Alpha. Section 2 contains an elementary introduction to classical Alpha graphs and a description of the role of Jordan's Curve Theorem in this setting. In section 3 we embark on the adventure of studying existential (Alpha) graphs on a number of different surfaces. In this paper we will consider only the representation of propositional formulas, leaving the general study of the transformation rules on surfaces as an open problem. In section 4 we slightly enter into the logical interpretation of mappings between surfaces. Finally, in section 5 we draw some conclusions and also pose a few suggestions for future research.

2. Alpha Graphs and Jordan's Curve Theorem on the plane

The basic components of the Alpha graph system are, in Peirce's terminology, the *sheet of assertion*, the propositional *letters*, and *cuts* that are simple, closed curves. In any Alpha graph drawn on the sheet there are finitely many letters or cuts, and this components never touch each other although a cut may appear around other components, including other cuts. An *area* is a portion of the sheet limited by cuts, and an area is *even* or *odd* according to the number of cuts drawn around it.

The basic interpretation of Alpha graphs consists in two conventions. Firstly, writing a letter means to *assert* the proposition it stands for; writing two letters together means to assert both; in general, drawing a graph means to assert it. On the other hand, enclosing a letter or a graph means to *negate* the content of the cut. In this way, we obtain the graphs of the basic propositional connectives shown on Figure 1.

The rules of transformation for Alpha graphs are the following:

1. **Erasure in even.** On an even area, any graph may be erased.
2. **Insertion in odd.** On an odd area, any graph may be drawn.
3. **Inward iteration.** Any graph may be iterated on its own area or inside any cut contained in it, as long as this cut is not part of the iterated graph.
4. **Deiteration from the outside.** Any graph whose occurrence could be the result of iteration may be erased.

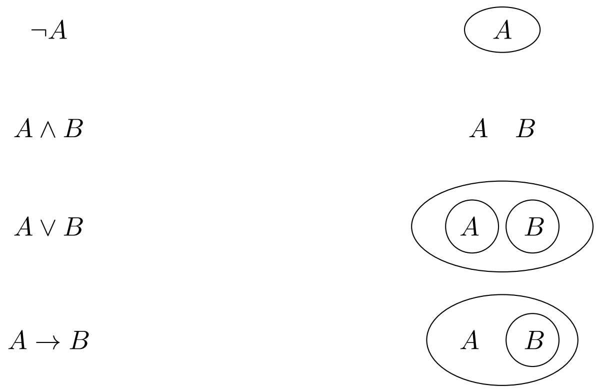

FIGURE 1. Alpha graphs for propositional connectives.

5. **Double cut.** A double cut, consisting of two nested cuts without letters or cuts in the area between them, may be drawn or erased around any graph on any area.

These rules are sufficient for the complete development of propositional logic in a graphical way. For instance, Figure 2 shows a diagrammatic proof of *modus ponens*. For more details about existential graphs, both Alpha, Beta and Gamma, for Peirce's views on assertion, and for different proofs of the equivalence between Propositional logic and Alpha graphs, see [1, 2, 4, 10, 11, 12, 13, 14].

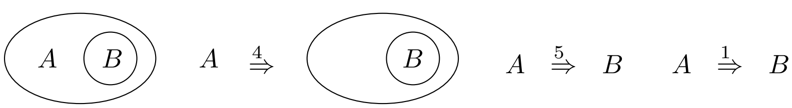

FIGURE 2. Proof of *modus ponens* by means of Alpha graphs.

Alpha graphs can be defined in a formal way as topological objects. As a model for the sheet of assertion we take the Cartesian plane $\mathbb{R}^2$ with its usual structure; a cut is a simple closed curve or *Jordan curve*, that can be defined as a continous injective function from the unit circle to the plane; at last, we represent a letter as a function (which is always continuous and injective) from the unit set to the plane. An *Alpha pre-graph* is an injective sum of a finite set of these functions, here the injectivity means that the different cuts do not touch each other nor the letters. Finally, an Alpha graph is an isotopy class of Alpha pre-graphs [9].

The celebrated Jordan's Theorem states that every Jordan curve divides the plane into two connected components, one of them bounded (the "inside") and the other unbounded (the "outside"), and that every point of the curve is adherent to both regions [3, 5]. From this result it follows that n disjoint Jordan curves divide the plane into $n+1$ connected components, n of them bounded and just one unbounded (the "outside").

In the framework of existential graphs, a first consequence of Jordan's Curve Theorem is that the logic on the sheet of assertion has two truth values. Since the cuts correspond to Jordan curves and the areas are exactly the connected components of the complement of the curves, a cut divides the sheet into just two areas specifying as many truth values. But a deeper effect of Jordan's Theorem is that n cuts define $n+1$ areas and that, among all these areas, there is one which is easily distinguishable. This is the unbounded component, which conventionally is associated with the truth value *True*. Hence, all other areas are automatically assorted into *True* or *False* according to their separation from the unbounded area by an even or odd number of cuts. This is another way to state the idea, basic to the theory of existential graphs, that an area is even or odd according to the number of cuts that enclose it. This parity is essential in the application of the rules of Erasure and Insertion. Moreover, the way of counting the areas from the outmost, *i.e.* the only unbounded one, provides a natural *orientation* in the structure of the graphs which was called by Peirce *endoporeutic* [11]. At its turn, this orientation is crucial in the use of the rules of Iteration and Deiteration.

In the clearly intended interpretation of existential graphs, the sheet of assertion relates to an intuitive notion of truth since every graph drawn on it is automatically asserted. A cut constitutes a rupture with this truth, but on the sheet as a whole it remains obvious with which of the two regions the truth is related. Hence, in a natural way, in this system there is one unique or "absolute" truth, given by the only unbounded component. Also, there is only one kind of negation, expressed by a cut, that ascribes to the enclosed component the truth value which is contrary to the value of the component where the cut is drawn.

3. Shifting towards other surfaces

Even though Jordan's Theorem generalizes to higher dimensions, it also makes sense to ponder its validity on different surfaces because the definition of a Jordan curve in this setting is straightforward. Here a *surface* is a topological 2-manifold [7, 8], which includes but is not restricted to differential manifolds and also Riemann surfaces. If we regard a surface as a generalized sheet of assertion, then we can view a Jordan curve on it as the cut of a certain system of Alpha graphs. Depending on the surface's geometry, these new existential graphs allow the interpretation of some or all propositional connectives. An adequate adaptation of the transformation rules gives rise to a deductive system or "logic" that we may reckon as been distinctive of the selected surface.

Just like in Geometry, on every surface we will find a local logic and a global one. In the local case, since the manifold has dimension 2, every point has an open neighborhood which is homeomorphic to the plane $\mathbb{R}^2$. If we restrict our attention to Jordan curves that are completely contained in such an open set, thus considering only graphs which are "small" compared to the whole surface, all diagrams on this subset behave like they were drawn on the plane. In some varieties the open sets homeomorphic to the plane fall apart into bounded an unbounded ones. If the open set is unbounded then Jordan's Curve Theorem holds exactly as on the plane, hence the definition of even and odd areas resembles completely the original Alpha graphs and the logic is classical. If the open set is bounded in some sense, it is anyway homeomorphic to $\mathbb{R}^2$ and its boundary corresponds to the point at infinity. In this way, it suffices to assign the truth value *True* to the boundary and then again we can regard the "small" graphs just as if it were plane graphs. In both cases the value *True* is attached to what is "far away".

***Remark* 3.1.** On any surface, every open set homeomorphic to $\mathbb{R}^2$ has a *local logic* given by a system of Alpha graphs that is equivalent to the Alpha graphs on the plane.

Hence, the local logic on any surface is the two-valued classical logic. Furthermore, the veritable interest in studying existential graphs on surfaces lies in its *global* logic, which according to the selected manifold most assuredly has interesting and meaningful differences with classical logic.

In the next paragraphs we will ponder the attributes of such a global logic on some particular surfaces.

3.1. The sphere

On this surface, Jordan's Theorem states that a Jordan curve divides the sphere into two connected components, both of them bounded [6]. Therefore, n disjoint Jordan curves divide it into $n + 1$ connected and bounded components, see Figure 3.

Since there is no natural way to highlight one spherical component over the rest, the global logic on the sphere lacks any "absolute" truth. Even if this logic is two-valued, given a certain area it is altogether unfeasible to decide whether it is *True* or *False*. This implies that if two graphs are drawn on the same component of the sphere we may not regard this juxtaposition as a conjunction, again because there is no way to determine whether the graphs are both *True* or both *False*. The only thing that can be said with certainty is that these graphs are *congruent* or *equivalent* in some sense. On the contrary, if the graphs are drawn on two adjacent components they are *incongruent* or *opposite*, but again it is impossible to label any of them as *True* or *False*.

Regarding to some form of the double negation principle, we can extend the above conventions to more curves between the components. Thus, in the Alpha graphs on the sphere two areas are defined to be *equivalent* if they are

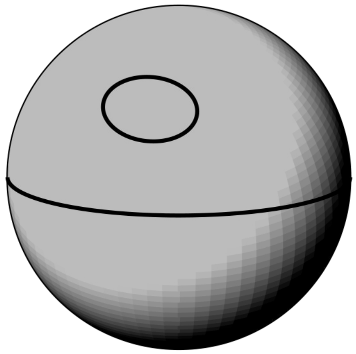

FIGURE 3. Jordan curves on the sphere.

separated by an even number of cuts, and *opposite* if they are separated by an odd number. About this two relations it would be feasible to decide, in a metalogical way, whether they are true or false about given areas and about graphs contained in them. In some sense this relations are consistent with the usual connectives biconditional $\leftrightarrow$ and exclusive disjunction $\veebar$, which are complementary to each other in classical logic.

Thus, the global logic on the sphere might merely express this two relations, equivalence and oppositeness, as well as the interplay between them. Hence this logic may correlate to the segment of classical propositional logic given by two connectives, biconditional and exclusive disjunction. Although it seems a rather poor logic, its generality and symmetry may turn out to be powerful.

With respect to the rules of transformation it seems reasonable to keep the freedom to draw or erase a double cut. About all other rules it is clear that they can not maintain their original form because on the sphere there are no even or odd areas, and there is no endoporeutic direction. Most certainly the rules should be thought of in a relative way.

Some of the above findings are true for any closed surface, *i.e.* for any compact 2-manifold without boundary, regardless of its genus. Of course, the surface should be connected.

***Remark* 3.2.** The global logic on a connected closed surface has no "absolute" truth and, consequently, the areas have no evenness nor endoporeutic orientation.

3.2. The cylinder

We consider a right circular cylinder unbounded in both directions, or in a homeomorphic presentation, a bounded circular cylinder without borders. On this surface there are two essentially different kinds of Jordan curves: those that are contractible to a point (by means of a homotopy on the cylinder) and those that are not contractible because they circle the cylinder once, see Figure 4. Contractible curves correspond to the trivial element of the fundamental group, while non-contractible give rise to a generator of this group which is infinite cyclic, or isomorphic to $\mathbb{Z}$. In another sense, contractible curves are contained in an open subset homeomorphic to the plane, but non-contractible are not contained in any such a subset. Finally, contractible curves divide the cylinder into two connected components, one bounded and another unbounded, while non-contractible divide the surface into two components which are both unbounded. This last distinction comprises Jordan's Curve Theorem on this manifold: on the cylinder a Jordan curve divides the surface into two connected components. These components can be one bounded and one unbounded, or else both unbounded.

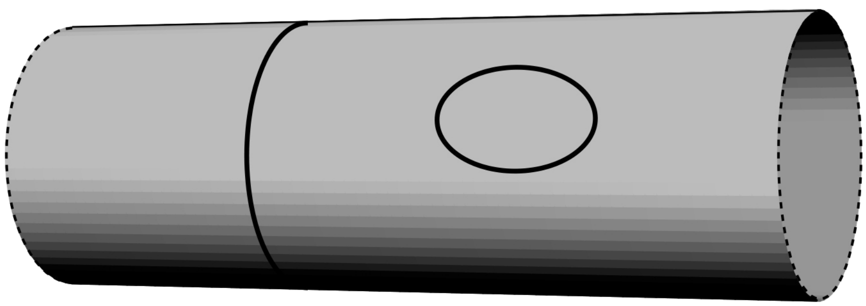

FIGURE 4. Jordan curves on the cylinder.

The global logic on the cylinder has two "relative" truths given by the two unbounded ends, and two kinds of negations associated to the two types of cuts. A contractible cut means the negation of the bounded area or inside with respect to the unbounded or outside. Hence, in any graph, an area might be considered *True* or *False* regarding any of the relative truths if it is separated from the appointed end by an even or odd number of cuts. On the other hand, a non-contractible cut that turns around the cylinder is a negation that sets up an antithesis between the relative truths. Since the cuts on the cylinder are intrinsically different, it seems reasonable that the negations represented by them have also distinct behavior. In case that the negation associated with non-contractible cuts satisfies the double negation principle, then an even number of this cuts drawn on the cylinder indicate that the relative truths are equivalent, while an odd number of non-contractible cuts mean that they are opposite. Under this circumstances, if the relative truths are compatible then every area has only one truth value and the logic becomes classical; on the contrary, if they are opposite then every area is *True* with respect to one relative truth and *False* with respect to the other.

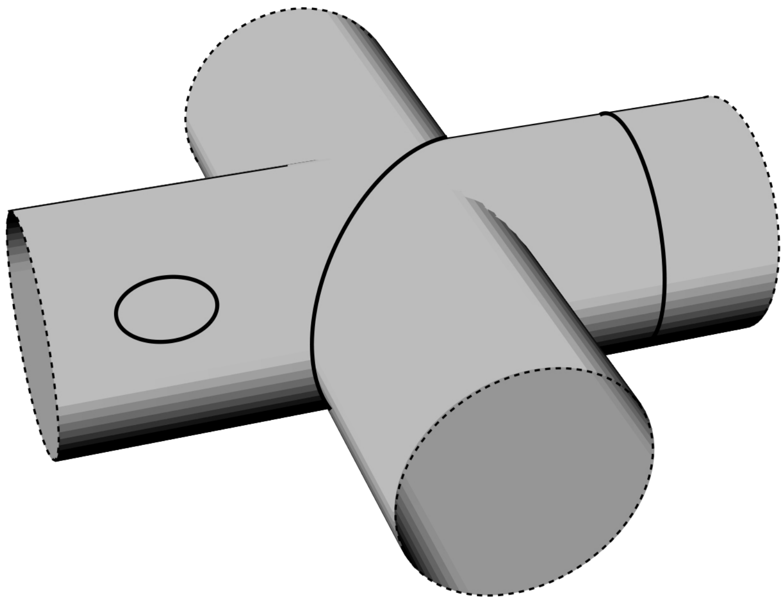

FIGURE 5. Jordan curves on a cylinder with four unbounded ends.

We can generalize this construction. A cylinder with three unbounded ends is like a three-way pipe joint; a cylinder with four unbounded ends is the intersection of two common cylinders, like a four-way fitting, see Figure 5. More generally, we can consider a surface with n unbounded cylindrical ends. The logic determined by Alpha graphs on such a manifold has n relative truths. The non-contractible cuts around this surface describe the compatibilities between these truths, while a contractible cut just expresses the negation of the bounded inside with respect to the unbounded outside. Every area will have n truth values, each one with respect to a certain relative truth: it is *True* or *False* whether the area is separated from the appointed end by an even or odd number of cuts. Again, if the negation given by the non-contractible cuts satisfies the double negation principle and all relative truths are equivalent, then each area has an unique truth value and the logic becomes classical.

3.3. The Möbius strip

We consider the Möbius strip without border, *i.e.* as a non-bounded surface. Any contractible Jordan curve on this well-known surface divides it into two connected components, one bounded and another unbounded, just as on the cylinder and on the plane. This curves are contained in an open set homeomorphic to the plane and they determine the trivial element of the fundamental group. But on the Möbius strip there are also non-contractible Jordan curves over the length of the band. This "long" curves are not contained in any open set homeomorphic to $\mathbb{R}^2$ and originate a non-trivial element of the fundamental group, which again is isomorphic to $\mathbb{Z}$. But there are two different types of this curves along the Möbius strip: in one case, the curve traces two turns along the surface and divides it again into two connected components, one bounded and another unbounded (see Figure 6); in the other case, the curve draws a single turn along the Möbius strip and *does not divide* it into two components, (see Figure 7). This last type of curve determines a generator of the fundamental group and is completely contained in the bounded component delimited by any curve of the preceding class. In brief, a Jordan curve on the Möbius strip divides it into two connected components or not. If it divides the surface, one component is bounded and the other is not.

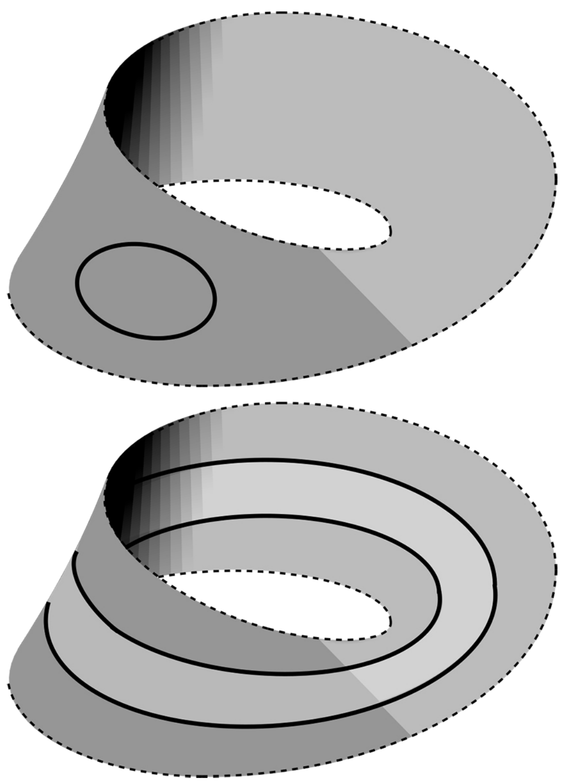

FIGURE 6. Separating Jordan curves on the Möbius strip.

In the global logic on the Möbius strip there is just one "absolute" truth, but there are at least two kinds of negations. As always, a cut on the strip that divides it into a bounded area and an unbounded one expresses the negation of the bounded inside with respect to the unbounded outside. So far, this logic seems close to classical logic, although it remains to clarify the meaning of the non-contractible cuts. But on this surface there are also cuts that do not mark out two different areas, which is a novel and quite strange element of this graphs. What does it mean, in the framework of existential graphs, to draw a cut that does not divide the sheet of assertion? One fact we can point out in this juncture is that, since different cuts are always disjoint in existential graphs, if on the Möbius strip we draw a cut that does not divide it then we may not trace another with the same peculiarity.

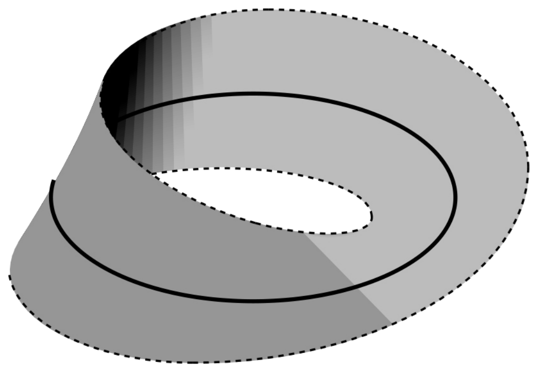

FIGURE 7. A non-separating Jordan curve on the Möbius strip.

3.4. The torus

On this surface there are again two different kinds of Jordan curves. Contractible curves are contained in an open set homeomorphic to the plane, generate the trivial element of the fundamental group and divide the torus into two connected and bounded components. On the other hand, there are non-contractible curves not contained in any open set homeomorphic to $\mathbb{R}^2$, that give rise to a non-trivial generator of the fundamental group (which is isomorphic to $\mathbb{Z} \times \mathbb{Z}$), and *do not divide* the torus into two components. For this last kind of curves there are various different possibilities: a Jordan curve may circle the "hole" of the torus once, or it may circle the "body" of the torus once, or describe a spiral around the body of the torus, or else a spiral around the hole, see Figure 8. These options correspond to elements $(1, 0)$, $(0, 1)$, $(1, m)$ and $(n, 1)$ in the fundamental group, and each of them generates an infinite cyclic subgroup. Thus, a Jordan curve divides the torus into two connected components, both bounded, or else does not divide it, for which there are multiple possibilities.

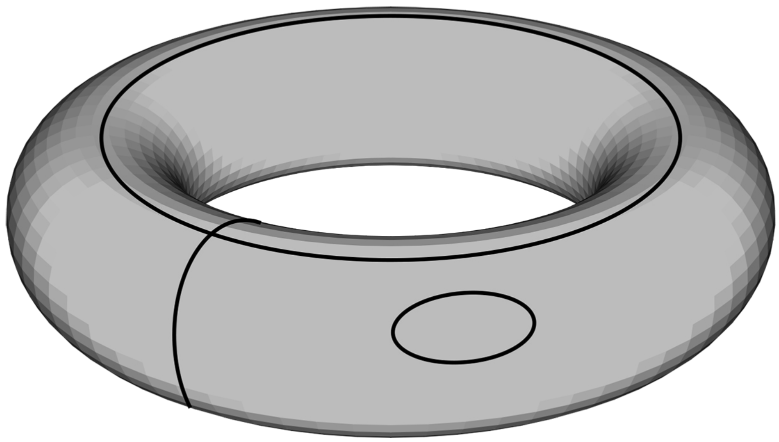

FIGURE 8. Jordan curves on the torus.

The global logic on the torus has no "absolute" truth by Remark 3.2, and there are at least two kinds of negations. A cut on the torus that divides it into two components expresses the incompatibility or antagonism between the separated areas, in a similar way to the logic on the sphere. But again, a cut on the torus that does not divide it into two areas is an element whose logical meaning is not yet clear. In contrast to the previous surface, on the torus it is possible to draw essentially different cuts that do not divide it into two areas. Anyway, if a graph on the torus has such a non-contractible cut then all non-contractible cuts are of the same type: for instance, if we draw a cut circling the tube of the torus then we can not draw any around the hole nor any spiral, and all drawn cuts should circle the tube.

3.5. Punctured surfaces

According to Remark 3.2, on the closed surfaces like the sphere and the torus there is no "absolute" truth because it is not feasible to highlight in a natural way some area that stands for the truth value *True*. One way to achieve this is punching holes in the surface by removing a finite set of points.

By the well-known stereographic projection, a sphere with just one point removed is homeomorphic to the plane, and hence the global logic on the sphere punctured at one point is exactly classical logic. This fact constitutes a quite interesting turnabout because now classical logic materializes as a particular case of a more fundamental system. First, classical logic is expressed by existential graphs on the plane; then, the graphs are developed on the sphere defining there a new logical system; finally, since the plane can be obtained from the sphere, classical logic becomes a special case of another system.

The sphere punctured at two points is homeomorphic to the cylinder, and more generally the surface with n unbounded cylindrical ends, mentioned in section 3.2, is just a sphere punctured at n different points. In this way, the logic on the sphere attains still greater relevance because various logics mentioned arise from this one.

A torus punctured at one point may be thought of as a cylinder that finishes as a torus at one of its ends (Figure 9), hence this surface has only one unbounded end and the logic on it has an "absolute" truth. Besides the contractible cuts and the cuts on the torus that do not divide the surface, now there are cuts circling the cylinder that express a negation of the whole surface of the torus with respect to the absolute truth.

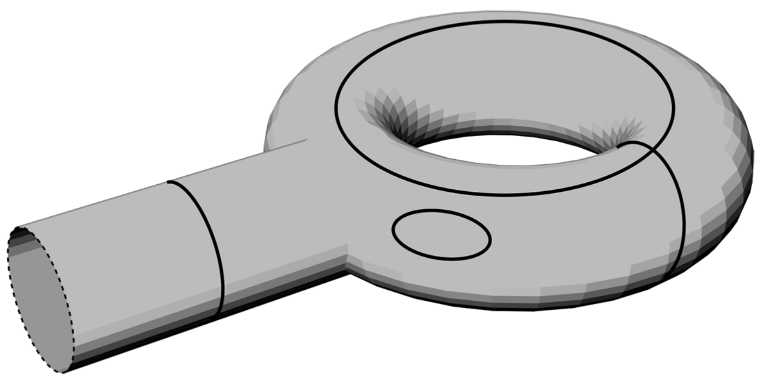

FIGURE 9. Jordan curves on the punctured torus.

In this way it seems a prospective project to study in a systematic way the logic intrinsic to all closed surfaces, oriented and unoriented, and then each of them punctured at any finite number of points.

4. Mappings

Now that it appears feasible to assign a natural logic to any surface, a mapping between different surfaces attains a logical connotation as a translation (in the sense of rendering) from one logic into another.

The class of mappings chosen between certain surfaces depends naturally on the structure considered on the manifolds. For instance, we would consider continuous maps between topological manifolds, differentiable mappings between smooth manifolds, and holomorphic functions between Riemann surfaces. But this requirements may turn out not to be sufficient in order to translate any logical features, not even the local logic. Along the project put forward in this paper, the logic inherent to any surface is given by the Alpha graphs on it, and in turn the behavior of Alpha graphs on any manifold depends on the Jordan curves on the surface. Hence, in order that any mapping between surfaces somehow translates a surface logic, it should preserve Jordan curves besides any likely topological features associated with them.

If a mapping between surfaces restricts to a homeomorphism between open sets both homeomorphic to the plane, then all Jordan curves contained in one of these open sets translate faithfully into the other and, moreover, the inside and outside are preserved in all cases. This means that the logic given by the Alpha graphs is translated in a perfect way. Particularly local homeomorphisms, which include for instance all conformal maps, preserve the local logic.

***Remark* 4.1.** Any local homeomorphism between surfaces preserves the local logic intrinsic to both surfaces.

Some other suggestive examples of mappings are in place here. As mentioned earlier, the inverse of the stereographic projection is a homeomorphism and translates the logic of the plane into the logic of the sphere. On the other hand, complex meromorphic functions from the torus to the sphere correspond to double periodic functions, known as elliptic functions. In this way, the theory of elliptic functions may help to elucidate the logic associated to the torus.

5. Concluding remarks

By means of a very natural generalization of Peirce's existential graphs, any topological 2-manifold is furnished with an inherent logic. As stated above, the propositional part of this logic is given by the Alpha graphs on the surface, and this graphs depend on the Jordan curves on it. On many surfaces the behavior of this curves, and hence the behavior of the negation in this logic, is described by Jordan's Curve Theorem on the manifold. However, as might be expected, these considerations result in many open problems. In the first place, it is imperative to investigate more deeply into the logic on the mentioned surfaces, or at least, on certain families of manifolds.

Here we can already specify certain general issues. If we read the contractible Jordan curves as the usual negation, just as on the plane, how do we interpret non-contractible ones? And the curves that do not divide the surface into two components? These questions are, in turn, deeply related to the open problem of specifying the rules of transformation for the graphs on the various manifolds. For instance, it seems feasible that in any case a double cut, made up of two nested contractible cuts without signs between them, may be drawn or erased freely. But for the remaining rules we foresee serious difficulties. What does it mean that a certain area is even or odd? What does it mean that a graph is contained in a given area? Is there always an endoporeutic orientation?

The idea of developing existential graphs on surfaces may turn out to have connections with other topics of current interest in mathematics, like *dessins d'enfants* on Riemann surfaces, or the possibility to understand geometrically different types of negation.

Beyond many technical details and quite interesting connections, the line of inquiry suggested in this paper seems to have deep consequences in itself. Actually, by the course outlined here the *way of thinking* ascribed to a certain logic becomes associated very precisely to the global *form* of a specific surface. Perhaps, the form of our reasonings relies on the form of our living space.

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