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THE AHARONOV-BOHM EFFECT, CONTROVERSIAL FEATURES OF A LONG-STANDING DEBATE[a]

EL EFECTO AHARONOV-BOHM, ASPECTOS CONTROVERSIALES EN UN DEBATE DE LARGA DURACIÓN

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Artículo Invitado

ABSTRACT: In the so-called magnetic Aharonov-Bohm effect, a quantum interference pattern shift is produced when electrons move in a magnetic field-free region, thus in absence of forces. Analogous fringe shifts are observed in interference experiments even when electrons travel through a magnetic field and are thus affected by magnetic forces. Because of the vast dedicated literature covering this subject it could require a non trivial effort to attain a comprehensive overview. Therefore, attention has been addressed: i) to recall the theory, ii) to describe the basic aspects of the main experiments realized up today and, iii) to review the long-standing debate regarding the interpretation of the Aharonov-Bohm phase shift as a new quantum topological effect with no analogue in classical theory, or as an energy-related lag effect based on classical forces.

KEYWORDS: Aharonov-Bohm effect; electron interference; quantum phase difference; spatial lag shift.

RESUMEN: En el fenómeno denominado efecto magnético Aharonov-Bohm, se produce un desplazamiento del patrón de interferencia cuántico cuando los electrones se mueven en una región libre de campo magnético y, por lo tanto, en ausencia de fuerzas. También se observan cambios análogos en experimentos de interfe- rencia cuando los electrones viajan a través de un campo magnético, siendo afectados por fuerzas magnéticas. Debido a la vasta literatura especializada sobre este tema, se requiere un esfuerzo no trivial para abarcar el panorama completo. Así, en este artículo se ha concentrado la atención en los siguientes aspectos: i) rediscutir los fundamentos teóricos, ii) describir los aspectos básicos de los principales experimentos rea- lizados hasta ahora y, iii) revisar el ya largo debate sobre la interpretación del cambio de fase en el efecto Aharonov-Bohm como un fenómeno topológico cuántico sin analogía en la teoría clásica, o como un retardo relativo a la energía causado por la acción de fuerzas clásicas.

PALABRAS CLAVE: Efecto Aharonov-Bohm; interferencia de electrones; diferencia de fase cuántica; desplazamiento por retardo espacial.

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1. INTRODUCTION

In a well known article, Aharonov-Bohm, hereafter referred to as AB, called attention to the significance of the electromagnetic potentials in quantum theory (Aharonov & Bohm, 1959). In particular, they proposed an electron interference experiment in which a coherent electron beam is split into two coherent parts that are subsequently recombined to form an interference pattern. An infinitely long solenoid is inserted between the two beams. Electrons move in a multiply-connected field-free region since the magnetic field is completely confined inside the solenoid. However, the electron waves suffer a phase difference that can be observed in an interference pattern formed on a screen. The striking result is that such a phase shift is observed in spite of the fact that electrons are not directly affected by magnetic fields. It must be emphasized that the behaviour of electron waves enclosing a magnetic flux and brought to interfere was already mentioned by Franz (1939) and subsequently carefully discussed by Ehrenberg & Siday (1949). The magnetic AB effect has stimulated a wealth of scientific literature regarding the interpretation of the phase difference revealed in an interference pattern (for a review see Peshkin & Tonomura (1989), Olariu & Popescu (1985), Matteucci *et al.* (2003), Batelaan & Tonomura (2009)). As a consequence, textbooks and teaching papers report mainly about this effect with the aim of conveying a good sense of the physical significance of the vector potential (Shadowitz, 1988; Feynman *et al.*, 1965; Felsager, 1998; Matteucci & Pozzi, 1978; Iencinella & Matteucci, 2004; Giuliani, 2010). However, there are experiments, based on the considerations developed by Boyer (1973), that use the effect of a magnetic field which acts directly on electrons to generate observable phase shifting effects analogous to those taking place in the 'true' AB experiment. These phase shifts can be interpreted as classical lag effects.

In spite of the importance of electromagnetic potentials in modern physics, the so-called magnetic AB effect is presented, in most of the books dealing with electromagnetism and quantum physics, mainly from a theoretical point of view, (Shadowitz, 1988; Feynman *et al.*, 1965; Felsager, 1998), while experimental aspects are disregarded. Up today more than 400 articles have been published regarding the AB effects and more than 3000 citations witness the widespread interest (multidisciplinary physics, condensed matter, philosophical works, etc.) and the so-called secondary citations exceeds 65.000 citations. The scientific community has widely accepted the validity of the magnetic AB effect because it opens a new approach regarding the way we think about electromagnetic fields in quantum physics.

Here, a contribution is presented concerning the controversial discussion about the considerations which have stimulated a lively debate regarding the interpretation of the AB magnetic phase shift as a classical lag effect due to the presence of forces. The discussion is focused to recall only the main theoretical and experimental aspects and highlight the subtleties regarding the interpretation of the interference phase shifts.

2. THE MAGNETIC AND ELECTRIC AHARONOV-BOHM EFFECTS

To illustrate the historical development of the stimulating debate regarding the interpretation of the magnetic phase shift also the so called electric AB effect (Aharonov & Bohm, 1959) is briefly reviewed because it plays an important role to understand the considerations developed to explain the magnetic AB effect in terms of a direct interaction between charged particles and magnetic fields (for a detailed introduction to the electric AB effect and related experiments see Matteucci (2007)).

2.1. The magnetic AB effect

Let us consider a charged particle travelling in a region where the electromagnetic fields vanish. From a classical point of view, we expect that the charge motion remains unaffected. However, an interesting example is described regarding electrons moving through a region free from electric and magnetic fields but where only a confined magnetic flux is present (Ehrenberg & Siday, 1949; Aharonov & Bohm, 1959; Olariu & Popescu, 1985). Since there are no forces on the particle, a possible detectable effect can only be interpreted with a quantum mechanical approach. The Schroedinger equation, in fact, involves electromagnetic potentials. Aharonov and Bohm suggested the following interference experiment to demonstrate the role of the vector potential in quantum theory (Aharonov & Bohm, 1959). As shown in Figure 1, an electron beam EB is split, by the interferometer A, into two secondary coherent beams 1 and 2 that travel around the metal plate MP. These beams are brought to overlap and form an interference pattern on the detector D.

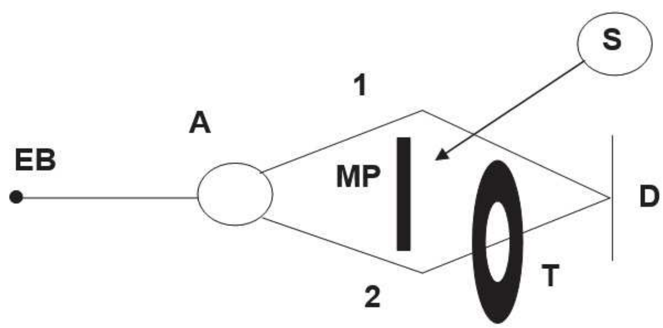

Figure 1: Schematic arrangement to reveal the magnetic AB effect.

As expected, the interference fringe distribution is symmetric with respect to the optical axis, where the principal maxima is located. Now, suppose that an infinitely long solenoid S (axis perpendicular to the drawing) or, alternatively, a perfect toroidal magnet T is placed behind MP as shown in Figure 1. These magnetic configurations are devised to allow electrons to move in field free regions. In Figure 1, if the toroid magnet T is used, the electron beam 1 moves outside T while beam 2 travels through its central hole. From a classical point of view, no changes in the interference pattern is expected because no magnetic field is ever acting on electrons. However, electron waves that pass on both sides of this localized magnetic flux suffer a phase difference that can be observed in the interference pattern. This phase shift was described first by

Eherenberg and Siday in their considerations on the meaning of the refraction index in electron optics and, subsequently, by AB in their discussion on the significance of the electromagnetic potentials in quantum theory. According to AB, a phase shift can be observed in the interference fringes. Although electron waves move in a field free region their phase is affected by the vector potential which is non vanishing along the closed curve described by the two paths 1 and 2 connecting the points A and D, Figure 1. By applying the Stokes theorem to this closed path or to any other closed path between the points A and D which include the confined magnetic flux, the resulting phase difference $\Delta\varphi$ is given by:

$$\Delta\varphi = \frac{2\pi}{h} e\Phi \tag{1}$$

where e, h and Φ are the electron charge, the Planck's constant and the magnetic flux embraced by electron trajectories. By increasing the magnetic flux, the phase difference may be changed from zero to an arbitrary multiple of 2π without ever washing out the interference fringes.

2.2. The electric AB effect

As shown in Figure 2, a coherent electron beam EB is split into two parts which are then chopped to form wave packets.

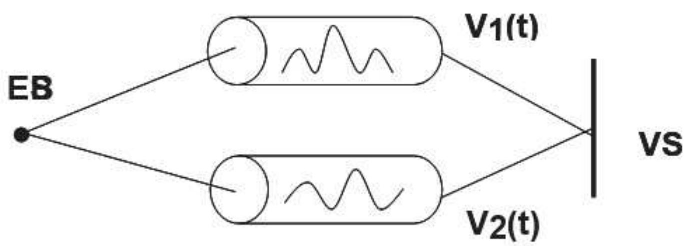

Figure 2: The electric AB experiment.

Subsequently, each part enters a long metal cylinder, the electric potential $V(t)$ of which is varied only when the electron wave packets are well inside the tubes. The tubes act as Faraday cages so that the group velocity of both wave packets is not changed because no force is ever exerted on electrons. The beams are then recombined on the viewing screen VS where an interference pattern is formed. AB demonstrated that only the electron phase velocity is affected so that a phase difference $\Delta\varphi$ is expected in the interference pattern only due to the action of the time-dependent scalar potentials $V_1(t)$ and $V_2(t)$ applied to the two cylinders:

$$\Delta\varphi = \frac{2\pi}{h} e \int V_1(t)dt - \frac{2\pi}{h} e \int V_2(t)dt \tag{2}$$

By increasing the potentials $V_1(t)$ and/or $V_2(t)$, the phase difference may be changed from zero to an arbitrary multiple of 2π without ever washing out the interference pattern.

These quantum effects, arising from both the magnetic and electric potentials, are periodic functions which are invariant to regular gauge transformations (for a detailed discussion see Olariu & Popescu (1985)). Therefore, interference experiments allow the direct testing of the underlying principle of gauge field theory.

2.3. Experimental methods

The first experiment to reveal the existence of the magnetic AB phase shift was carried out by Chambers using a magnetic whisker (Chambers, 1960). Subsequently, the phase shift was also observed using an electrostatic biprism as interferometry device (Möllenstedt & Düker, 1956). The biprism consists of a thin charged wire W (about 1μm diameter) flanked by two earthed metal plates, Figure 3.

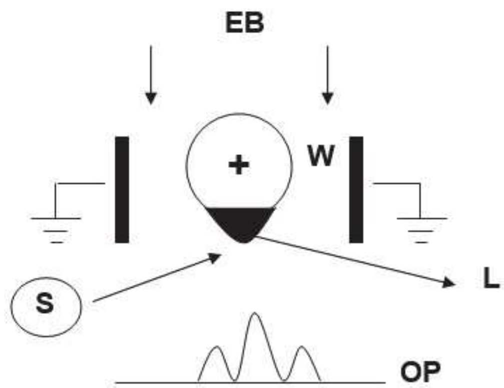

Figure 3: Working principle of an electrostatic biprism.

W splits the incoming electron beam EB into two parts that, without breaking the spatial coherence condition, travel on the left and right-hand sides of the wire. A positive potential applied to the fibre W produces a convergence of the electron trajectories and their overlapping in the observation plane OP where an interference pattern system is formed. We recall only the most significant experiments to show the different strategies adopted to localize the magnetic field in the interferometer (for a review see Peshkin & Tonomura (1989), Olariu & Popescu (1985)). With reference to Figure 3, they are grouped as follows: i) the magnetic field S, produced by a ferromagnetic whisker or by a thin solenoid is located in the geometrical shadow of the biprism wire (Möllenstedt & Düker, 1956; Möllenstedt & Bayh, 1962), ii) a thin iron whisker (Schaal *et al.*, 1966; Fowler *et al.*, 1961) or, alternatively, a superconductor hollow cylinder (Lischke, 1969) replaces the biprism fiber and, at the same time, acts as a source of magnetic field, iii) the shadow region of the wire W in Figure 3 is vacuum coated with a thin ferromagnetic layer L (Fowler *et al.*, 1961; Matteucci & Pozzi, 1978). The main rebuttal to all these experiments regards the fact that electrons move, respectively, through the stray field of the whiskers, solenoids, etc. so that the phase shifts could be produced by the effect of the magnetic leakage field on passing electrons. However, this is not the case. For example, it was demonstrated that the stray field of a slender bar of iron was not responsible of the phase difference observed in the interference pattern (Matteucci & Pozzi, 1978).

2.4. Further experiments related to the magnetic and electric AB effects

A different approach to explain the magnetic AB phase shift has been developed by Boyer. He started his considerations by taking into account either the electric or the magnetic AB effects (Boyer, 1973). First of all, Boyer demonstrated that a phase shift analogous to that observable for the AB time-dependent electric potential was expected in case the two tubes were held at a constant potential. As electrons enter the tubes, they experience classical electrostatic forces which cause a change of their speed. For non-relativistic particle motion, these small speed changes are, $\Delta V_1 = eV_1/p$ and $\Delta V_2 = eV_2/p$ where p, V_1, V_2 are, respectively, the electron momentum at an infinite distance from the tubes and the potentials of the two tubes. At the exit of the cylinders, the initial speeds of electrons are restored but the particles remain relatively displaced in the direction of motion. The electrostatic forces produce no net change of momentum or energy for the electrons but only a classical relative displacement Δz in the direction of motion. Therefore, the phase shift in the interference pattern was explained as a classical lag effect. This interpretation of Boyer was confirmed by the beautiful experiment carried out by Schmid (1984). He brought to interfere electrons passing through a tube held at a constant potential with electrons moving in a field free region. This experimental condition is analogous to the two-tube arrangement, provided one tube is grounded while the other is at a constant potential. By increasing the voltage to the tube of 160 μV, a phase shift of 2π was observed. With a phase shift of $2\pi \exp(7\times10^5)$ which corresponds to a coherent wave packet length l_c=0.46 μm, the washing out of the interference pattern was finally detected. In other words, the interference pattern is washed out when the two wave packets suffer a spatial lag larger than their length. It is curious to note that this beatiful experiment is almost completely disregarded in the literature.

Almost at the same time, two interference experiments were realized with electrons traveling through the electrostatic field of a bimetallic wire or, alternatively, of two wires having opposite charges (for a review of these experiments see Matteucci & Pozzi (1982), Matteucci *et al.* (1984), Matteucci & Pozzi (1985), Matteucci & Pozzi (1987), Matteucci *et al.* (1998)). In the present article, we report only about the case in which a macroscopic dipole acts as a phase shifting device. The dipole, which consists of two lines of opposite charges, is inserted between two coherent electron beams travelling along the z-axis perpendicular to the dipole axis. Subsequently, the two electron wave fronts are brought to overlap to form an interference pattern. In particular, it comes out that the revealed phase shift is due to a local interaction of the passing electrons with the electrostatic field of the dipole. The z-component of the electric field of the dipole, parallel to the electron trajectories, causes a spatial lag and the consequent phase difference revealed in the interference pattern (Matteucci & Pozzi, 1982; Matteucci *et al.*, 1984; Matteucci & Pozzi, 1985; Matteucci & Pozzi, 1987). An important point deserves to be underlined. The phase difference could depend on different lateral deflection undergone by particles passing on either side of the dipole. However, it was demonstrated that no sideway deflection of electrons was present. For this reason, although questionable, Matteucci and Pozzi, following the ideas of Aharonov and Bohm and taking into account the considerations of Boyer regarding the phase shift produced with time-independent potentials, named the outcomes of their experiments as ‘electrostatic’ AB effects. These results were reconsidered by Boyer who confirmed that the phase shift had a clear explanation in terms of an electrostatic lag effect associated with the component of the electric force of the dipole acting parallel to the beam trajectories (Boyer, 2002). Subsequently, Hilbert, Caprez and Batelaan performed an experiment similar to that of Matteucci and Pozzi in which the two lines of dipoles were realized with two parallel, oppositely biased wires (143 μm diameter), separated by about 1 mm (Hilbert *et al.*, 2011). The flight times of electron pulses, emitted by a source and passing by the two wires, were measured with different voltages on the wires. Corresponding time delays were observed consistent with the predictions of Matteucci, Pozzi and Boyer.

3. INTERPRETATION OF INTERFERENCE PATTERN SHIFTS: QUANTUM TOPOLOGICAL VIEW OR CLASSICAL-LAG POINT OF VIEW?

Let us now introduce the considerations regarding the phase shifts, arising from either static electric fields or confined magnetic fluxes, interpreted as classical lag effects due to the presence of forces. In a number of papers, Boyer discussed the possibility to explain also the AB magnetic phase shift as a classical lag effect (Boyer, 1973) (see a full list of references in Boyer (2002), Tonomura *et al.* (1986)). When an electron is passing along side a solenoid, a change in the energy of the electromagnetic field arises due to the overlapping of the magnetic field produced by that electron and the magnetic field of the solenoid. In analogy to the two-tube electrostatic experiment, a relative classical lag $\Delta L = e\Phi/mv$ takes place for electrons (m=mass, v=velocity) passing on opposite sides of the solenoid thus producing exactly the same phase shift $\Delta\varphi = 2\pi\Delta L/\lambda$, ($\lambda$ is the electron wave length), of equation 1 revealed in the experiments described before (Möllenstedt & Düker, 1956; Fowler *et al.*, 1961; Boersch *et al.*, 1962; Möllenstedt & Bayh, 1962; Schaal *et al.*, 1966; Lischke, 1969; Matteucci & Pozzi, 1978; Matteucci *et al.*, 2003).

The difference between the phase shifts observable in the ‘true’ electric and magnetic AB effects must be highlighted with respect to the phase shifts arising from the corresponding classical lag effects. In the ‘true’ electric AB effect, the phase velocities of the two wave packets are changed by the potentials $V_1(t)$ and $V_2(t)$ applied to the two tubes, Figure 2. The group velocities of both wave packets are not affected because no force is present inside the cylinders. An analogous situation takes place in the ‘true’ magnetic AB effect. A local phase effect on the particle wave functions is produced while the group velocities remain constant. Differently, according to Boyer’s semi-classical interpretations, the electric and magnetic phase shifts arise as follows: i) in the electrostatic experiments the phase shift is due to electric forces on electrons that cause a change of the group velocity and a related spatial lag, while the phases of the wave packets are unaffected; ii) in the magnetic experiment the phase difference is the result of a spatial lag of the wave packets due to the action and reaction forces between the passing electrons and the solenoid. In both cases, the electric and magnetic forces, responsible of the observed phase shifts, are parallel to the electron beam trajectories.

In 1986, Tonomura and co-workers realized a new experiment with a tiny toroidal magnet (Tonomura *et al.*, 1986). According to the authors, the stray fields were completely excluded by a superconducting niobium cladding and a further copper layer completed the shielding from the electron wave. Electron holography method was used to reveal the phase difference between two electron beams, one passing outside the toroid and the other threading through the magnet central hole. This experiment seemed to remove all objections about the influence of magnetic leakage fields and unphysical character of an infinite solenoid of the AB experiment.

In 2002, further considerations were presented regarding the analogies for the interpretation of the phase shifts, in terms of forces, due to either the magnetic AB effect or the line of electric dipole experiment (Boyer, 2006). In particular, the solenoid was considered as a stack of current loops i.e., a pile of magnetic dipoles. The magnetic field arising from an electron traveling near the solenoid produces a magnetic field and a net force on the solenoid itself. It is assumed that the action of the passing electron on the solenoid and the force of the solenoid on that electron satisfy Newton's third law. This force causes a change of electron velocity along the direction of motion. A relative displacement of the two wave packets passing on both sides of the solenoid is generated and a phase difference is revealed in the interference pattern. As we have seen previously, this force based approach has also been used to explain the phase shift in an electron interference pattern caused by a line of electric dipoles placed between two interfering beams. Therefore, a parallel between magnetic and electric experiments has been sketched to interpret the AB quantum phase shifts in terms of electromagnetic forces.

Because the AB effects are fundamental in nature, it is important to consider a further basic feature. Electrons move in field free regions so that they are not accelerated and their group velocities do not change. In case of the magnetic AB effect, the vector potential outside the solenoid produces a change of the canonical and not of the kinetic momentum of electrons. Therefore the resulting relative phase shift $\Delta\varphi = 2\pi e\Phi/h$ is non-dispersive, i.e. it is independent of the velocity of the incoming electrons. Therefore an experiment able to demonstrate this particular feature would, according to Zeilinger: '…represents rather convincing evidence of the special nature of the AB effect' (Zeilinger, 1986). An experiment has been realized following this line of thought (Matteucci *et al.*, 2003). Electron holography was used to reveal the phase changes of the electron waves passing on both sides of a long magnetic nano-wire replacing the solenoid of the AB experiment. Electrons at five different velocities in the range (160000-210000) km/s were used. The phase differences resulted constant within experimental uncertainties of 3 % on the whole range of electron velocities. However, it must be emphasized that also in this experiment electrons propagate through the leakage field of the nano-wire and the conditions required by the 'true' magnetic AB effect are not fulfilled.

It must be pointed out that the experiment in which the solenoid is replaced with a toroidal magnet, whose flux is confined with a metal layer of superconducting material, is considered as an unquestionable evidence of the existence of the magnetic AB effect (Tonomura *et al.*, 1986). Due to the screening effect of the superconductor shield, it was assumed that the electrons passing on both sides of the toroidal magnet might travel in field free regions. This experiment has been regarded as the best test to discriminate if the observed phase shift can be accounted for in terms of vector potential or of forces according to the Aharonov-Bohm's, the Liebowitz's or the Boyer's assertions respectively (Aharonov & Bohm, 1959; Liebowitz, 1965; Boyer, 2002). However, it is interesting to report the Boyer's considerations (Boyer, 2002) applied to the Lischke and Tonomura experiments with superconductors (Lischke, 1969; Tonomura *et al.*, 1986): 'Experiments have been performed which attempt to shield the solenoid from the magnetic fields of the passing electrons. The persistence of the phase shift despite the presence of conducting materials shielding the solenoid has been interpreted as excluding the possibility of a phase shift based upon classical electromagnetic forces'. 'Actually, although a superconductor expels the magnetic field lines of a time-independent magnetic field in the Meissner effect, a superconductor acts similarly to a normal metal for high frequency fields. The erroneous point of view regarding the screening role of conducting materials appears on page 426 of the review by Olariu & Popescu (1985) and on page 123 of the review by Tonomura (Peshkin & Tonomura, 1989). However, most physicists are unaware that magnetic velocity fields penetrate conducting ohmic materials in a fashion which is completely different from the skin depth penetration of electromagnetic wave fields, (see Matteucci & Pozzi (1978) in Boyer's paper (Boyer, 2002)). Thus according to classical electromagnetic theory, the attempts to screen out the electromagnetic fields of the passing charges have actually been ineffectual'. To support his interpretation, Boyer reported to have derived, '…an invariant time integral which has precisely the correct form to account for the Aharonov-Bohm phase shift as an energy-related lag effect based on classical forces' (see Boyer (2002) and references therein).

At this stage, the main question is: does the magnetic AB phase shift arise from a classical lag effect in analogy with the electrostatic effects discussed previously? Caprez, Barwick and Batelaan devised an apparatus consisting of two identical solenoids connected by high permeability magnet iron bars to form a square magnetic toroid (Caprez *et al.*, 2007). This arrangement, together with a magnetic cylindrical shield, placed between the two coils along the electron's propagation direction reduces unwanted magnetic stray fields. These experimenters attempted to measure if an electron beam, passing between the two solenoids, suffered a time delay. If electrons do not suffer any time delay there is no velocity change and hence no spatial lag, therefore the AB effect is interpreted according to the presently accepted quantum theory. No time delay was measured, thus signalling absence of force in two conditions: i) when the magnetic flux was changed by varying the solenoid current and, ii) when the magnetic shield was removed. Moreover, it was ascertained that the metal shield, which contained the solenoid, was not responsible of any detectable delay effect that could potentially explain the AB effect. According to their experimental results, Caprez, Barwick and Batelaan, concluded that the AB effect could not arise by forces on electrons passing a macroscopic solenoid and the explanation of the AB effect in terms of a classical spatial lag had thus to be rejected (Caprez *et al.*, 2007). However, it must be emphasized that the two experiments carried out by Hilbert *et al.* (2011) and Caprez *et al.* (2007), show a different behaviour for electrons passing through an electrostatic field and through a square toroidal magnet, respectively. For electrons passing the lines of dipole a delay time was observed and this is exactly what is needed for the interpretation of the phase shifts recorded in previous electrostatic experiments (Matteucci & Pozzi, 1982; Matteucci *et al.*, 1984; Matteucci & Pozzi, 1985; Matteucci & Pozzi, 1987; Matteucci *et al.*, 1998). On the other side, the time lag expected between charged particles passing on opposite sides of the solenoid was of the order of 10^{-7} seconds for the largest current values through the coils (Caprez *et al.*, 2007). No time delay was found although the experimental accuracy was sufficient to reveal the presence of forces necessary to explain the AB effect. However, following the magnetic experiment of Caprez, Barwick and Batelaan, Boyer proposed again a classical interpretation of the AB effect. He considered a magnetic dipole oriented along the z-axis and moving at a given velocity along the z-axis (Boyer, 2008). This dipole moves toward a number of equally- spaced electrons located in a plane $z = 0$. These electrons are forced to move on a circular loop of radius r centred on the z-axis. The particles on the ring are accelerated and in turn put a force back on the incoming magnetic moment. This example is used, in a tight analogy, to describe the interaction of a charged particle passing a solenoid (Boyer, 2008). According to the Boyer's interpretation, the passing charges produce an acceleration of the solenoid electrons and these react with a force on the passing charge. The friction between particles of the solenoid play an important role: '…if the frictional forces (solenoid resistance) is large, we do not expect to find the time lag because the acceleration of the solenoid charges will be small and hence the back forces on the passing charge will be small' (Boyer, 2008). On the contrary, when frictional forces are very small the solenoid electrons will produce a force back on the passing charges and, as a consequence, an associated time lag. Moreover, Boyer (2008) compares the time lag (10^{-7} s) expected in the experiment of Caprez *et al.* (2007) and the much sorter passage time of electrons past the solenoid in the experiment of Möllenstedt & Bayh (1962) (about 10^{-13} s). The passage time of electrons in the Möllenstedt and Bay's experiment '…is not much longer than the collision time 10^{-14} sec in the Drude model for conductivity of a metal, …Thus it is possible that the conservation of energy involving magnetic fields holds in the short-time regime where the Möllenstedt and Bay's experiment were performed, yet would not hold for the much longer passage times for the slower electrons passing the much larger solenoid in the experiment Caprez, Barwick, and Batelaan (Caprez *et al.*, 2007). It is worthwhile adding that the passage time of electrons from a thin magnetic filament as that used in the early versions of the AB experiments, is even much shorter, about 10^{-17} s (Matteucci & Pozzi, 1978; Möllenstedt & Düker, 1956). Moreover, the time lag associated with the spatial lag between particles passing on opposite sides of this long magnetic filament is of the order of 10^{-19} sec.

It must be emphasized, however, that experiments are needed to support the theoretical interpretations of these phase shifts and to asses whether or not the AB phase difference results from the absence or presence of velocity changes for the passing particles. All this matter, therefore, remains poorly understood.

4. CONCLUDING REMAKS

We have collected together and compared the results of the 'thought experiment' suggested by Aharonov-Bohm with those of real experiments. While the 'true' electric AB effect has never been realized, the phase differences measured between interfering electrons travelling through static potential distributions have been clearly explained as spatial lag effects due to the presence of electric forces. The interpretation of the phase shift in terms of a lag effect due to classical electromagnetic forces or as a quantum effect taking place in absence of forces is much more controversial when electrons move through regions in which a localized magnetic flux is present. According to Boyer, the way to give a definite answer to this problem is to realise an experiment in which a large magnetic flux is enclosed between two interfering beams so that the phase shift generated is larger than that associated with the wave packet length. If the group velocity of electrons moving on opposite sides of the solenoid is changed, then the interference pattern is washed out provided a sufficient large magnetic flux is enclosed between the electron beams. On the contrary, in case the magnetic flux is increased indefinitely without ever destroying the interference pattern it means that the phase shift does not involve velocity changes of electrons. According to Boyer, it is precisely this phase shift that will represent a clear, conclusive demonstration of the quantum topological nature of the magnetic Aharonov-Bohm effect. This experiment, however, has not yet been realized.

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References

Aharonov, Y. & Bohm, D. (1959). Significance of electromagnetic potentials in quantum theory. *Physical Review,* 115, 485-491.

Batelaan, H. & Tonomura, A. (2009). The Aharonov-Bohm effects: variations on a subtle theme. *Physics Today,* 38-43.

Boersch, H.; Hamisch, H. & Grohmann, K. (1962). Experimenteller Nachweis der Phasenverschiebung von Electronenwellen durch das magnetische Vektorpotential II. *Zeitschrift für Physik A Hadrons and Nuclei,* 169(**2**), 263-272.

Boyer, T. (1973). Classical electromagnetic deflections and lag effects associated with quantum interference pattern shifts: considerations related to the Aharonov-Bohm effect. *Physical Review D,* 8(**6**), 1679-1693.

Boyer, T. (2002). Semiclassical explanation of the Matteucci-Pozzi and Aharonov-Bohm phase shifts. *Foundations of Physics,* 32, 41-49.

Boyer, T. (2006). Darwin-Lagrangian analysis for the interaction of a point charge and a magnet: considerations related to the controversy regarding the Aharonov-Bohm and Aharonov-Casher phase shifts. *Journal of Physics A: Mathematical and General,* 39(**13**), 3455-3477.

Boyer, T. (2008). Comment on experiments related to the Aharonov-Bohm phase shift. *Foundations of Physics,* 38, 498-50.

Caprez, A.; Barwick, B. & Batelaan, H. (2007). Macroscopic test of the Aharonov-Bohm effect. *Physical Review Letters,* 99(**21**), 210401.

Chambers, R. (1960). Shift of an electron interference pattern by enclosed magnetic flux. *Physical Review Letters,* 5, 3-5.

Ehrenberg, W. & Siday, R. (1949). The refractive index in electron optics and the principles of dynamics. *Proceedings of the Physical Society. Section B,* 62, 8-21.

Felsager, B. (1998). *Geometry, particles, and fields.* New York: Springer Science & Business Media.

Feynman, R.; Leighton, R. & Sands, M. (1965). The Feynman Lectures in Physics. Addison-Wesley Pub. Co.

Fowler, H.; Marton, L.; Simpson, J. & Suddeth, J. (1961). Electron interferometer studies of iron whiskers. *Journal of Applied Physics,* 32(**6**), 1153-1155.

Franz, W. (1939). Elektroneninterferenzen im Magnetfeld. Verhandlungen der Deutschen Physikalischen Gesellschaft. 2, 65.

Giuliani, G. (2010). Vector potential, electromagnetic induction and ‘physical meaning’. *European Journal of Physics,* 31(**4**), 871-880.

Hilbert, S.; Caprez, A. & Batelaan, H. (2011). Experimental observation of time-delays associated with electric Matteucci-Pozzi phase shifts. *New Journal of Physics,* 13(**9**), 1-9.

Iencinella, D. & Matteucci, G. (2004). An introduction to the vector potential. *European Journal of Physics,* 25(**2**), 249-256.

Liebowitz, B. (1965). Significance of the Aharonov-Bohm Effect. *Il Nuovo Cimento,* 38(**2**), 932-950.

Lischke, B. (1969). Direct observation of quantized magnetic flux in a superconducting hollow cylinder with an electron interferometer. *Physical Review Letters,* 22(**25**), 1366-1368.

Matteucci, G. (2007). Quantum effects of electric fields and potentials on electron motion: an introduction to theoretical and practical aspects. *European Journal of Physics,* 28(**4**), 625-634.

Matteucci, G. & Pozzi, G. (1978). Two further experiments on electron interference. *American Journal of Physics,* 46(**6**), 619-623.

Matteucci, G.; Missiroli, G. & Pozzi, G. (1982). A new electrostatic phase shifting effect. *Ultramicroscopy,* 10(**3**), 247-251.

Matteucci, G.; Missiroli, G.; Porrini, M. & Pozzi, G. (1984). Recent observations on a new electrostatic phase shifting effect. *Proceedings of the International Symposium on Foundations of Quantum Mechanics in the Light of New Technology* (pp. 39-43). Kamefuchi et al. eds. Physical Society of Japan, Tokio.

Matteucci, G. & Pozzi, G. (1985). New diffraction experiments on the electrostatic Aharonov-Bohm effect. *Physical Review Letters,* 54(**23**), 2469-2472.

Matteucci, G. & Pozzi, G. 1987. The Aharonov-Bohm effect and the quantum interference phase shift due to an electrostatic field. *Quantum Uncertainties* (pp. 297-306). Edited by W. M. Honig, D.W. Kraft & E. Panarella. Plenum Publishing Corporation.

Matteucci, G.; Missiroli, G. & Pozzi, G. (1998). Electron holograms of long-range electrostatic fields. *Advances in Imaging and Electron Physics, 99,* 171-240.

Matteucci, G., Iencinella, D. & Beeli, C. (2003). The Aharonov-Bohm phase shift and Boyer's critical considerations: new experimental results but still an open subject?. *Foundations of Physics,* 33(**4**), 577-590.

Möllenstedt, G. & Düker, H. (1956). Beobachtungen und Messungen an Biprisma-Interferenzen mit Elektronenwellen. *Zeitschrift für Physik,* 145(**3**), 377-397.

Möllenstedt, G. & Bayh, W. (1962). Messung der kontinuierlichen Phasenschiebung von Elektronenwellen im kraftfeldfreien Raum durch das magnetische vektorpotential einer Luftspule. *Die Naturwissenschaften,* 49(**4**), 81-82.

Olariu, S. & Popescu, I. (1985). The quantum effects of electromagnetic fluxes. *Review of Modern Physics,* 57(**2**), 339-436.

Peshkin, M. & Tonomura, A. (1989). The Aharonov-Bohm effect. *Lecture Notes in Physics,* 340.

Schaal, G.; Joensson, C. & Krimmel, E. (1966). Weitgetrennte kohaerente Elektronen-Wellenzuge und Messung des Magnetflusses $\Phi_0 = h/e$. *Optik,* 24(**5**), 529-538.

Schmid, H. (1984). Coherence length measurement by producing extremely high phase shifts. *Proceedings of the Eight European Congress on Electron Microscopy* (pp. 285-286). Budapest, Hungary. Edited by A. Csnady, P. Roehlich and D. Szabò (Programme Committee of the Congress, Budapest).

Shadowitz, A. (1988). *The electromagnetic field.* USA: Dover Publications Inc.

Tonomura, A.; Osakabe, N.; Matsuda, T.; Kawasaki, T.; Endo, J.; Yano, S. & Yamada, H. (1986). Evidence for Aharonov-Bohm effect with magnetic field completely shielded from electron wave. *Physical Review Letters,* 56(**8**), 792-795.

Zeilinger, A. (1986). Generalized Aharonov-Bohm experiments with neutrons. Fundamental aspects of quantum theory. Edited by *V. Gorini and A. Frigerio,* 311-318.